Trig. inequality with tangents-5454 SSMJ
https://www.linkedin.com/feed/update/urn:li:activity:6786788689923198976
5454. Proposed by Arkady Alt , San Jose , California, USA.
Prove that for integers k, l any any $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$, the following inequality holds:

$$k^2 \tan\alpha + l^2 \tan\beta \geq \frac{2kl}{\sin(\alpha+\beta)} - (k^2 + l^2)\cot(\alpha+\beta).$$

Solution by proposer.
Since $\cos\alpha, \cos\beta, \sin(\alpha+\beta) > 0,$ we obtain

$$k^2 \tan\alpha + l^2 \tan\beta \geq \frac{2kl}{\sin(\alpha+\beta)} - (k^2 + l^2)\cot(\alpha+\beta) \Leftrightarrow$$

$$k^2(\tan\alpha + \cot(\alpha+\beta)) + l^2(\tan\beta + \cot(\alpha+\beta)) \geq \frac{2kl}{\sin(\alpha+\beta)} \Leftrightarrow$$

$$\frac{k^2\cos\beta}{\cos\alpha\sin(\alpha+\beta)} + \frac{l^2\cos\alpha}{\cos\beta\sin(\alpha+\beta)} \geq \frac{2kl}{\sin(\alpha+\beta)} \Leftrightarrow (k\cos\beta - l\cos\alpha)^2 \geq 0.$$

Equality occurs when $\frac{k}{\cos\alpha} = \frac{l}{\cos\beta}$.

Remark.
The problem was sent to the magazine in the following formulation:
Prove that for any $\alpha, \beta \in \left(0, \frac{\pi}{2}\right)$ and any real k, l the following inequality holds:

$$k^2 \tan\alpha + l^2 \tan\beta \geq \frac{2kl}{\sin(\alpha+\beta)} - (k^2 + l^2)\cot(\alpha+\beta).$$